Morlet Wavelets and Wavelet Convolution

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FEBRUARY 18, 2014
Last chapter: Fourier Transform and Fast Fourier Transform

- Cannot tell changes in frequency structure over time
- Violates Assumptions of Fourier Analysis—EEG data are not stable over time
- Solution? Do not use a temporally stable wave for convolution
  - Fourier Transform- KERNEL: Sine Waves (no temporal information extracted because they have time constant oscillations)
  - Wavelets are not time domain constant – a KERNEL that can extract both frequency and time
- Still have assumption of temporal stability to deal with, but now the EEG signal only has to be stable when the wavelet looks like a sine wave.
Morlet Wavelet

- Morlet wavelet—
- A sine wave that is “Windowed” (i.e., multiplied point by point) by a Gaussian
- Can use other wavelets, but not all are well-suited
- Must taper to zero at both ends and have a mean value of zero
Morlet (Gabor) wavelet
Why use wavelets?

- No time variance
- Strong time variance
- Less frequency precision
- Box car - time and frequency - rough edges
- Wavelet - time and frequency - tapered edges
Make a Wavelet!

- Parameters to set

  - `srate = 500; % sampling rate in Hz`
  - `f = 10; % frequency of the sine wave in Hz, wavelet “peak” frequency`
  - `time = -1:1/srate:1; % time, from -1 to 1 second in steps of 1/sampling-rate`
Make a sine wave

- `sine_wave = exp(2*pi*1i*f.*time);`
Make a Gaussian!

- \( s = \frac{6}{2\pi f} \); (width, SD of the Gaussian)
- \( \text{gaussian\_win} = \exp(-\text{time.}^2/(2s^2)) \);
- “M” is deleted - not relevant for eeg

\[
\text{wavelet} = \text{sine\_wave} \times \text{gaussian\_win};;
\]

Exp="complex” = no amplitude adjustment???

f = frequency

Value of 6 set here for “n” - number of wavelet cycles
(tradeoff between temporal and frequency precision)
What happens when we change n?

- Time-frequency tradeoffs in precision
- Why?
“Family of Wavelets”

- Similar to decomposition with sine waves of different frequencies in FFT, time-frequency decomposition involves wavelets of different frequencies.

- To make a family you change the frequency of the sine wave while leaving other parameters unchanged.

- UNLIKE FFT: You can specify and use as many wavelets as you want.
Theoretical and Practical Limits

- You cannot use frequencies slower than your epochs – if you have 1 secs of data, you cannot analyze below 1 hertz. It’s best to have several cycles of activity, analyze 4 hertz and faster.
- Cannot use frequencies above the Nyquist frequency (one-half the sampling rate).
- Frequency smoothing occurs so that close frequencies will produce identical results (too many will increase computation time) - between 15 to 30 frequencies, spanning 3 to 60 hz should be sufficient for your “family”.
Another point

- The frequency information you obtain at one time point is a weighted sum of the frequency information at surrounding time points.

- In interpreting time-frequency results, each time point is an estimate of instantaneous activity influenced by neighboring activity.
A family (12.4 figure)
Color equals amplitude

Movie = 1 line
Another way to think of it: Band pass filtering

- Convolution with Wavelets at certain frequencies is like band pass filtering
- Reflects activity at peak frequency of the wavelet (6 hertz in next slide), but also activity from a weighted combination of surrounding frequencies (3hz to 9 hertz)
Wavelet looks like Gaussian

Convolution in time domain is multiplication in the frequency domain.
Make Figure 12.5
Imagine this! There are limitations...

- “Real” morlet wavelets act as bandpass filters, but in time-frequency analysis, we need power and phase information too...
- Convolution with the morlet wavelet depends on phase offsets.
- Without help from more dimensions (imaginary ones), we would have to line up the wavelet so it was at zero degree lag with the EEG data each time.
Dot product depends on relative phase

Wavelet at 10 hertz

One-cycle sine wave at 10 hertz
What to do? Complex wavelets!

- Can not only bandpass filter the data to extract frequency, but allows for time-frequency power and phase information